$$z = u v^2 w^3$$

$$u = \sin x \qquad v = -\cos x \qquad w = e^x$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial z}{\partial w} \cdot \frac{\partial w}{\partial x}$$

$$g(x,y) = f(x^2+y^2)$$ Zkusme $f = Id$, $f(x) = x$

$\frac{\partial g}{\partial x}(x,y)$, $\frac{\partial g}{\partial y}(x,y) = ?$ $\quad f(z) =$ neznáme

Vnitřní fce $\varphi(x,y) = x^2+y^2$

Jsme v situaci V30, kde $d=1$, $k=2$.

F z V30 je zde g.

$$\frac{\partial g}{\partial x}(x,y) = \frac{\partial f}{\partial z}(\varphi(x,y)) \cdot \frac{\partial \varphi}{\partial x}(x,y) =$$

$$= \underbrace{f'(x^2+y^2)}_{1} \cdot 2x \quad = 2x \; ✓$$

$$\frac{\partial g}{\partial y}(x,y) = \frac{\partial f}{\partial z}(\varphi(x,y)) \frac{\partial \varphi}{\partial y}(x,y) = \underbrace{f'(x^2+y^2)}_{1} \cdot 2y \quad (*)$$

$$\frac{\partial^2 g(x,y)}{\partial y \partial x} = \frac{\partial}{\partial y}\left(f'(x^2+y^2) \cdot 2x\right) =$$

$$= 2x \cdot \frac{\partial}{\partial y}\left(f'(x^2+y^2)\right) =$$

$$= 2x \cdot 2y \cdot \underbrace{f''(x^2+y^2)}_{= (Id)'' = 0} \quad = 0 \; ✓$$

↑ analogicky jako $(*)$, jen přidejte jednu čárku.

$$\frac{\partial^2 g(x,y)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial g(x,y)}{\partial x}\right) = \frac{\partial}{\partial x}\left(f'(x^2+y^2) \cdot 2x\right)$$

$$= \frac{\partial}{\partial x}\left(f'(x^2+y^2)\right) \cdot 2x + f'(x^2+y^2) \cdot \frac{\partial}{\partial x}(2x) =$$

$$= \underbrace{f''(x^2+y^2)}_{0} \cdot 2x \cdot 2x + \underbrace{f'(x^2+y^2)}_{1} \cdot 2 \quad = 2 \; ✓$$

$z = u\sqrt{1+v^2} \qquad u = e^{2x} \qquad v = e^{-x}$

$\tilde{z}(x) = e^{2x}\underbrace{\sqrt{1+e^{-2x}}}$

$\tilde{z}'(x) = \left(\qquad \right)' \quad \ldots \quad$ 1. SEMESTR

Pomocí řetízkového pravidla

$z(u,v) = u\sqrt{1+v^2} \qquad u(x) = e^{2x} \qquad v(x) = e^{-x}$

$\tilde{z}(x) := z(u(x), v(x))$

$\tilde{z}'(x) = \frac{\partial z}{\partial u}(e^{2x}, e^{-x}) \cdot \frac{\partial u(x)}{\partial x} + \frac{\partial z}{\partial v}(e^{2x}, e^{-x}) \cdot \frac{\partial v(x)}{\partial x}$

$\frac{\partial z}{\partial u}(u,v) = \sqrt{1+v^2} \qquad \frac{\partial z}{\partial v}(u,v) = u \cdot \frac{2v}{2\sqrt{1+v^2}} = \frac{uv}{\sqrt{1+v^2}}$

$= \sqrt{1+e^{-2x}} \cdot 2e^{2x} + \frac{e^{x}}{\sqrt{1+e^{-2x}}} \cdot (-e^{-x}).$

$$\boxed{\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}}$$

1c) $z = \sin u \cos v \qquad u = (x-y)^2 \qquad v = x^2 - y^2$

$\frac{\partial z}{\partial x}(x,y) = \cos u \cos v \cdot 2(x-y) + (-\sin u \sin v) \cdot 2x$

$= \cos(x-y)^2 \cos(x^2-y^2) \cdot 2(x-y) -$

$- \sin(x-y)^2 \sin(x^2-y^2) \cdot 2x$

$$\boxed{\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}}$$

$\frac{\partial z}{\partial y}(x,y) = \cos u \cos v \cdot (-2(x-y)) + (-\sin u \sin v) \cdot (-2y)$

$= \cos(x-y)^2 \cos(x^2-y^2) \cdot 2(y-x) + \sin(x-y)^2 \sin(x^2-y^2) \cdot 2y$